HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

2012

YEAR 12

TASK 3

Examiners ~ S. Cupac, P. Biczo, S. Faulds

General Instructions

- Reading time – 3 minutes.
- Working time – 40 minutes.
- Attempt **all** questions
- Board approved calculators and MathAids may be used
- This examination must **NOT** be removed from the examination room

- **Section A** consists of three (3) multiple choice questions worth 1 mark each. Fill in your answer on the multiple choice answer sheet provided.
- **Section B** requires all necessary working to be shown in every question. This section consists of three (3) questions worth 10 marks each. Marks may not be awarded for careless or badly arranged work.
  Each question is to be started in a new answer booklet. Additional booklets are available if required.

SECTION A – 3 multiple choice questions (1 mark each)

Question 1

$\int (2x+1)^5 \, dx =$

A $\dfrac{(2x+1)^6}{12}$

B $\dfrac{(2x+1)^6}{6}+C$

C $\dfrac{(2x+1)^6}{12}+C$

D $\dfrac{(2x+1)^5}{10}$

Question 2

Two ordinary dice are rolled. The score is the sum of the numbers on the top faces.
What is the probability that the score is **not** 12?

A $\dfrac{1}{36}$ **B** $\dfrac{1}{18}$ **C** $\dfrac{17}{18}$ **D** $\dfrac{35}{36}$

Question 3

For a particular value of *x*, say $x = a$, the minimum value of *y*, where *y* is expressed in terms of *x*, occurs when:

A both $y' = 0$ and $y'' > 0$ for $x = a$

B both $y' = 0$ and $y'' < 0$ for $x = a$

C both $y'' = 0$ and $y' > 0$ for $x = a$

D both $y'' = 0$ and $y' < 0$ for $x = a$

SECTION B

Question 4 (10 marks) Use a SEPARATE writing booklet **Marks**

(a) Find: $\int \frac{2x^3 - 5}{x^2} dx$ **2**

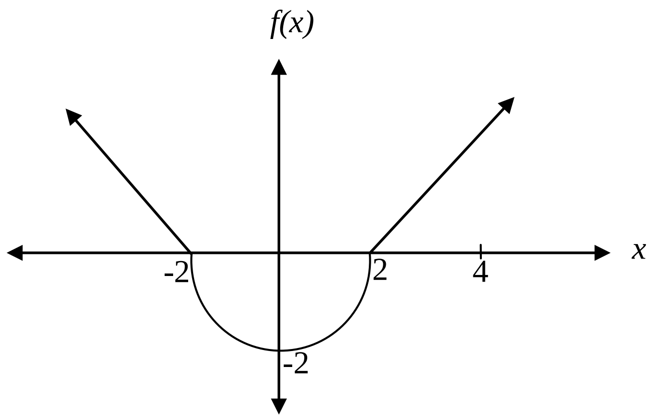
(b) Evaluate the definite integral: $\int_{-3}^{3} 9 - x^2 \, dx$ **2**

(c) Find the area enclosed by the curve $y = \sqrt{x - 5}$, the y axis and the lines $y = 1$ and $y = 3$. **2**

(d) (i) A piece-meal function $y = f(x)$ is defined as follows:

$$f(x) = \begin{cases} -x - 2, & x < -2 \\ -\sqrt{4 - x^2}, & -2 \le x \le 2 \\ x - 2, & x > 2 \end{cases}$$

The graph of this function is shown below:



Find the **exact** value of the integral $\int_{-2}^{4} f(x) dx$. **2**

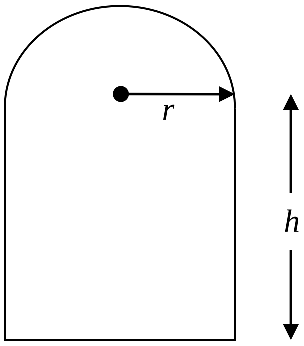
(ii) What is the area under the curve $y = f(x)$ between $x = -2$ and $x = 4$? **1**

(iii) Comment upon the similarities/differences in your calculations and answers to parts (i) and (ii). **1**

Question 5 (10 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the equation of the curve passing through the point $(2,-1)$ with gradient function $f'(x)=3x^2-4x+1$. **2**

(b) A window consists of a semi-circle of radius *r* metres sitting on top of a rectangle with height *h* metres.



(i) If the perimeter of the window is 7 metres, show that $h=\dfrac{7-2r-\pi r}{2}$ **1**

(ii) Show that the area of the window is given by $A=7r-\dfrac{1}{2}\pi r^2-2r^2$ **2**

(iii) Prove that the maximum possible area occurs when $r=\dfrac{7}{4+\pi}$ **3**

(iv) Find the maximum area in simplest fractional form. **2**

Question 6 (10 marks) Use a SEPARATE writing booklet **Marks**

(a) In a particular school the student population consists of 43% male and 57% female. Two students are selected at random to take part in a survey.

(i) Draw a probability tree to show all possible outcomes. **1**

(ii) Find, correct to two decimal places, the probability that both students are of different sexes. **2**

(b) In a herd of sheep, the probability of selecting a black sheep is approximately 1 in 15.

(i) What is the probability of not selecting a black sheep in each of three consecutive selections? **1**

(ii) How many consecutive selections must be made for it to be 90% certain that a black sheep will be selected? **3**

(c) When the Australian Hockey team of 32 members plays a game, they consume liquid for hydration. Some players drink only water, some players drink only Greatorade and some players drink both. In this team there are 24 players who drink water and 27 players who drink Greatorade:

(i) Show the liquid preferences of the Australian Hockey team in a Venn diagram. **1**

(ii) How many players drink both water and Greatorade? **1**

(iii) If one team member is selected at random, find the probability that they drink Greatorade but not water. **1**

This page has been left blank.

PART A ANSWER SHEET

- Detach this sheet and use it to mark the answers to the questions in Part A
- Mark the answer by shading the letter that matches with the correct answer
- If you make a mistake, draw a cross through the incorrect answer

Name: ___________________________

Class: _______________

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D

Year 12 Mathematics Task 3 Examination 2012

Question No. 4 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
H8	(a) $\int \frac{2x^3-5}{x^2}dx$ $= \int 2x - \frac{5}{x^2}dx$ $= x^2 + \frac{5}{x} + c$	**2 marks** Correct solution. **1 mark** Correctly splits expression into two fractions.
H8	(b) $\int_{-3}^{3} 9 - x^2 dx$ $= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$ $= (27-9) - (-27+9)$ $= 36$ *or* $\int_{-3}^{3} 9 - x^2 dx$ $= 2\int_{0}^{3} 9 - x^2 dx$ $= 2\left[9x - \frac{x^3}{3}\right]_{0}^{3}$ $= 2\left[(27-9) - 0\right]$ $= 36$	**2 marks** Correct solution. **1 mark** Determines the correct primitive function.
H8	(c) $y = \sqrt{x-5}$ *ie.* $x = y^2 + 5$ $A = \int_{1}^{3} y^2 + 5dy$ $= \left[\frac{y^3}{3} + 5y\right]_{1}^{3}$ $= (9+15) - \left(\frac{1}{3} + 5\right)$ $= \frac{56}{3}$ units^2	**2 marks** Correct solution. **1 mark** Determines the correct function to be integrated, using notation correctly.
H8	(d) (i) Using areas under curve to calculate the integral: $\int_{-2}^{4} f(x)dx = -\frac{1}{2} \times \pi \times 2^2 + \frac{1}{2} \times 2 \times 2$ $= 2 - 2\pi$	**2 marks** Correct solution. **1 mark** Determines one of the two areas correctly.
H8	(ii) Area under curve: $A = \left\lvert \int_{-2}^{2} f(x)dx \right\rvert + \int_{2}^{4} f(x)dx$ $= \frac{1}{2} \times \pi \times 2^2 + \frac{1}{2} \times 2 \times 2$ $= 2\pi + 2$ units^2	**1 mark** Correct answer.
H8	(iii) Both areas and integrals can be calculated by considering area enclosed between the curve and the *x*-axis, however, areas below *x*-axis must be considered as negative values when evaluating integrals.	**1 mark** Correct answer, demonstarating understanding of the concepts involved.

Year 12 Task 3 Mathematics Examination 2012

Question No.5 Solutions and Marking Guidelines

Outcomes Addressed in this Question

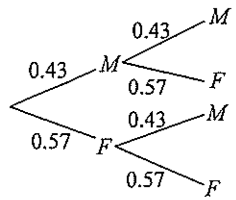
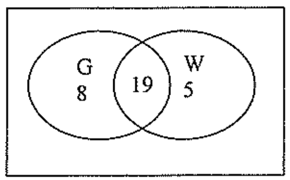
H6 Uses the derivative to determine features of the graph of a function

Outcome	Solutions	Marking Guidelines
H6	a)(i) $f'(x) = 3x^2 - 4x + 1$ $\therefore f(x) = x^3 - 2x^2 + x + c$ $(2,-1)$ lies on it. $\therefore -1 = 2^3 - 2\times 2^2 + 2 + c$ $\therefore c = -3$ $\therefore f(x) = x^3 - 2x^2 + x - 3$	2 marks: correct solution 1 mark: significant progress towards correct solution
	b) (i) Perimeter $= \frac{1}{2}$ circumference $+ 2h + 2r$ $\therefore 7 = \pi r + 2h + 2r$ $2h = 7 - 2r - \pi r$ $\therefore h = \frac{7 - 2r - \pi r}{2}$ $\therefore$ stationary points at $x = -2$ and $x = 1$	1 mark: correct solution
	(ii) Area = area of semicircle + rectangle $= \frac{1}{2}\pi r^2 + 2rh.$ $= \frac{1}{2}\pi r^2 + 2r\frac{(7 - 2r - \pi r)}{2}$ $= \frac{1}{2}\pi r^2 + 7r - 2r^2 - \pi r^2$ $\therefore A = 7r - \frac{1}{2}\pi r^2 - 2r^2$	2 marks: correct solution 1 mark: significant progress towards correct solution
	(iii) $\frac{dA}{dr} = 7 - \pi r - 4r$ For a maximum to occur $\frac{dA}{dr} = 0.$ Solving $7 - \pi r - 4r = 0$ $7 = \pi r + 4r$ $7 = r(\pi + 4)$ $r = \frac{7}{\pi + 4}$ To verify it is a maximum, $\frac{d^2A}{dr^2} = -\pi - 4 = -(\pi + 4) = -7.14...$ As $\frac{d^2A}{dr^2}$ is negative, the curve is concave down and hence a maximum when $r = \frac{7}{\pi + 4}$.	3 mark: correct derivative and solution 2 mark: substantially correct solution 1 mark: significant progress towards correct solution

	(iv) Maximum area when $r = \frac{7}{\pi+4}$ Substituting into $A = 7r - \frac{1}{2}\pi r^2 - 2r^2$, $\text{Area} = 7(\frac{7}{\pi+4}) - \frac{1}{2}\pi(\frac{7}{\pi+4})^2 - 2(\frac{7}{\pi+4})^2$ $= \frac{49}{\pi+4} - \frac{49\pi}{2(\pi+4)^2} - \frac{98}{(\pi+4)^2}$ $= \frac{98(\pi+4)}{2(\pi+4)^2} - \frac{49\pi}{2(\pi+4)^2} - \frac{196}{2(\pi+4)^2}$ $= \frac{98\pi+392-49\pi-196}{2(\pi+4)^2}$ $= \frac{49\pi+196}{2(\pi+4)^2}$ $= \frac{49(\pi+4)}{2(\pi+4)^2}$ $= \frac{49}{2(\pi+4)}$ units^2	2 marks: correct solution 1 mark: progress towards correct solution

Outcomes Addressed in this Question

H4 – expresses practical problems in mathematical terms based on simple given models
H9 – communicates using mathematical language, notation, diagrams and graphs

Outcome	Solutions	Marking Guidelines
H9	**a) (i)** 	**(1 mark)** correct probability tree
H4	**(ii)** $P(\text{different sexes})$ $= (0.43\times0.57)+(0.43\times0.57)$ $= \frac{2451}{5000}$ $= 0.4902$ $= 0.49$ (2 d.p)	**(2 marks)** correct solution with rounding. **(1 mark)** substantial progress towards correct solution
H4	**(b)(i)** $P(\overline{BBB}) = \frac{14}{15}\times\frac{14}{15}\times\frac{14}{15}$ $= \frac{2744}{3375}$	**(1 mark)** correct answer.
H4	**(ii)** $P(\text{a black sheep selected}) = 1 - P(\text{no black sheep})$ $= 1-\left(\frac{14}{15}\right)^n$ $1-\left(\frac{14}{15}\right)^n = 0.9$ $\left(\frac{14}{15}\right)^n = 0.1$ $n\log\left(\frac{14}{15}\right) = \log 0.1$ $n = \frac{\log 0.1}{\log\left(\frac{14}{15}\right)}$ $n = 33.34$ $\therefore$ 34 consecutive selections must be made.	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution.
H9	**c)(i)** 	**(1 mark)** correct venn diagram.
H4	**(ii)** 19 players drink both	**(1 mark)** correct answer
H4	**(iii)** $P(\text{only Greatorade}) = \frac{8}{32} = \frac{1}{4}$	**(1 mark)** correct answer.